

I. Définition

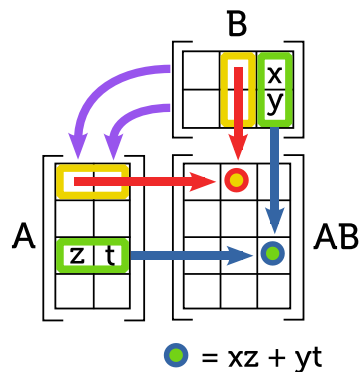
$$\begin{array}{l}
 f : \begin{array}{c} E \\ \mathbb{R}^n \end{array} \rightarrow \begin{array}{c} F \\ \mathbb{R}^p \end{array} \\
 \text{bases : } \mathcal{B} = (e_1, \dots, e_n) \quad \mathcal{B}' = (\varepsilon_1, \dots, \varepsilon_p)
 \end{array}
 \left| \begin{array}{l}
 \vec{X} = \sum_{j=1}^n x_j e_j \quad f(\vec{X}) = \sum_{j=1}^n x_j f(e_j) \\
 \vec{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \rightarrow f(\vec{X})
 \end{array} \right. \quad \boxed{f(e_j) = \sum_{i=1}^p \alpha_{i,j} \varepsilon_i}$$

$$\begin{array}{c}
 \begin{array}{ccccccc}
 & f(e_1) & & f(e_j) & & f(e_n) & \\
 & \downarrow & & \downarrow & & \downarrow & \\
 M = \begin{pmatrix} \alpha_{1,1} & \dots & \alpha_{1,j} & \dots & \alpha_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{i,1} & \dots & \alpha_{i,j} & \dots & \alpha_{i,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{p,1} & \dots & \alpha_{p,j} & \dots & \alpha_{p,n} \end{pmatrix} & \leftarrow \varepsilon_1 \\
 & & & & & & \leftarrow \varepsilon_i \\
 & & & & & & \leftarrow \varepsilon_p
 \end{array}
 \end{array}
 \left| \begin{array}{l}
 \boxed{f(\vec{X}) = MX} = \begin{pmatrix} x_1 \alpha_{1,1} + \dots + x_n \alpha_{1,n} \\ \vdots \\ x_1 \alpha_{p,1} + \dots + x_n \alpha_{p,n} \end{pmatrix} \\
 \quad \quad \quad \uparrow \quad \quad \quad \uparrow \\
 \quad \quad \quad x_1 f(e_1) \quad \quad x_n f(e_n)
 \end{array}
 \right.$$

II. Composée d'application

$$\boxed{M_{g \circ f} = M_g \times M_f}$$

III. Produit matriciel



IV. Dimension d'une matrice

$$\dim M_{n,p} = n \times p$$

V. Inversibilité d'une matrice de bijection

1. Théorème

$$\boxed{\begin{array}{l} \underbrace{A}_{= M_f} \text{ est inversible} \Leftrightarrow \exists \underbrace{B}_{= A^{-1} = M_{f^{-1}}} \mid AB = I_n \end{array}}$$

2. Méthode de calcul

a. Cas général

$$\left(\begin{array}{c|c} A & I_n \end{array} \right) \Rightarrow \dots \Rightarrow \left(\begin{array}{c|c} I_n & A^{-1} \end{array} \right)$$

b. Matrices 2×2

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad \boxed{A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}}$$

VI. Changement de base

Soient $\mathcal{B}(e_1, \dots, e_n)$, $\mathcal{B}'(\varepsilon_1, \dots, \varepsilon_n)$, $\mathcal{B}''$ différentes bases de E

1. Matrice de passage

$$\begin{array}{ccc} & \varepsilon_1 & \dots & \varepsilon_n \\ & \downarrow & & \downarrow \\ \boxed{P = P_{\mathcal{B} \rightarrow \mathcal{B}'} = \begin{pmatrix} & \dots & \\ \vdots & \ddots & \vdots \\ & \dots & \end{pmatrix}} & \leftarrow e_1 & & \\ & & & \vdots & \\ & & & \leftarrow e_n \end{array}$$

$$P_{\mathcal{B} \rightarrow \mathcal{B}''} = P_{\mathcal{B} \rightarrow \mathcal{B}'} \times P_{\mathcal{B}' \rightarrow \mathcal{B}''}$$

2. Passage de $\mathcal{B}$ à $\mathcal{B}'$

a. Vecteur

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{\mathcal{B}} \quad X' = \begin{pmatrix} x'_1 \\ \vdots \\ x'_n \end{pmatrix}_{\mathcal{B}'} \quad \boxed{X = PX'}$$

b. Matrice

$$M = \text{Mat}(f, \mathcal{B}) \quad M' = \text{Mat}(f, \mathcal{B}') \quad \boxed{M' = P^{-1}MP} \quad \underline{M'^x = P^{-1}M^xP}$$